Forwards & Futures

**Forwards**

# Forward Contracts

* **Agreement** initiated today to trade an asset on a fixed future date & fixed price
  + Fixed Future Date → **Maturity Date**
  + 
* The type of trade depends on the position of the Forward:
  + **Short Forward** → Agreement to Sell
  + **Long Forward** → Agreement to Buy
* Since there are no initial cashflows, the **Payoff & Profit** of a Forward contract is the same:
  + 
  + 

P •••uo uoqs 
WdS 
puewo-d 6u01 
PdS 
0 
/ 0 d 
0 
0 d 

## Cash Settlements

* Typically, Forward Contracts will result in **Physical Settlements** - an exchange of an Asset from one party to another
* For simplicity, some traders use **Cash Settlements** instead - transferring a cash amount equal to the profit from the "loser" to the "winner"
  + Person A agrees to buy one share from person B at $100 in one year
  + In one year, the market price is $150
  + Under Physical Settlements, person A will pay $100 to B in exchange for their share
  + Under Cash Settlements, person B will pay $50 to person to A to purchase the share in the market
* Cash Settlements are much **easier** to conduct & **reduces trading costs**

# 

* The most common way to derive the Forward Price is to use a **No-Arbitrage condition**
* Assuming no Arbitrage, if there was **no net investment** at time 0, then there **cannot be a net profit** at a future time T
* Using this logic, by forming a **portfolio of a Forward and an offsetting position at time 0**, we can manipulate the cashflows to determine the appropriate Forward Price

**Synthetic Forwards ("Offsetting Positions")**

* Both methods can be used to derive the Forward Price:
  + Using a Long Forward, we can offset it by **replicating a Short Forward** (Reverse Carry)
  + Using a Short Forward, we can offset it by **replicating a Long Forward** (Carry)
* As long as the original forward is being offset (Long & Short), the derivations will hold true

|  |  |
| --- | --- |
| **Cash & Carry** | **Reverse Cash & Carry** |
| Borrow Cash to **Carry the underlying** till expiration | Short Sell the underlying till expiration and Lend the cash (**Reverse**) |
| Sell the underlying at expiration | Buy the underlying at expiration |
| Pay Interest from the cash loan | Receive Interest from the cash loan |
| **Replicates a Long Forward** | **Replicates a Short Forward** |

Long Stock 
Spot Price 
Short Bond 
Synthetic Long Forward 
0 
Spot price 

## Underlying with No Dividends

* 
* Given the price of a contract with maturity at time T, we can find the price of a contract with a *different maturity* through the **time value of money**

Time T 
Long Forward 
Short Underlying 
Lend Cash 
Nel Cashflow 
Time O 
SO 
— SO 

### Stock VS Forward

* Assuming the Forward is correctly priced, there is no advantage of investing in a stock and forward
* 

**Underlying With Discrete Dividends**

* 
* 

Long Forward 
Short U nderlying 
Lend Proceeds 
Borrow Cash 
Net Cashtlow 
Time O 
so 
Time t 
—D 
Time T 
So erT 
D * era-t) 

**Determining the Dividends**

* Consider a situation where you calculate the forward price with **no dividends**
* But the **actual forward price is different** – this may NOT be a case of mispricing but rather **not accounting for dividends**
* By substituting in the two values, you can **calculate the accumulated value of the dividends**

**Bond Underlying**

* Note that a stock that pays discrete dividends is **similar to that of a Bond that pays regular coupons**
* The same formulas can be applied to determine the price of a forward contract which uses Bonds as the underlying asset

## Underlying with Continuous Dividends

* 
* Given the price of a contract with maturity at time T, we can find the price of a contract with a *different maturity* through the **time value of money**

Short Forward 
Long Underlying 
Borrow Cash 
Net Cashflow 
Time o 
—SO e—qT 
Time T 
ST 
— Fo 

### Why do we "ignore" the dividend cashflow?

* 
  + **Long Position** → Assume that the dividends are **reinvested**
  + **Short Position →** Assume that the owner **reinvests the dividends**
* 





Since continuous dividends are reinvested, we know that this change in value is due to the dividend.



# Value of a Forward Contract

* The **value of a position at a specific time** is the profit/loss of the position if the position were closed at that specific time
  + Closing a position means to take a **NEW equal and opposite position** with the same maturity
  + This cancels out the exposure of the original position, essentially "closing" it
* Assume we close a forward contract this way:
  + The obligation to buy and sell the underlying cancels out - the net cashflow will ALWAYS be the difference in Forward Prices
  + This is the value of the contract at time T will always be the difference in Forward Prices
  + Since this value is guaranteed, we can discount it to find the value of the contract at any time t
* 

Time O 
Time t 
Time T 
Long Forward 
Short New Forward 
Net Cashflow 

## Initial Value of a Forward Contract

* The initial value of any contract MUST be 0
* If you were to **open and close a position immediately**, it does not make sense for you to be able to make a straight profit/loss
* In another perspective, for any **trade to go through**, both parties must be **indifferent** to being the buyer/seller – if one was strictly better than the other, then no one would take the “worse” position

# 

* 
* From each of **their perspectives**:
  + 
  + 
* 
  + 
  + 
* Therefore, considering everything from the **perspective of the person issued at time 0**,
  + 
  + 
* Thus, it is just a **recalculation** using an **UPDATED spot price and time to maturity**

Issue original 
contract 
Original's time O 
Issue new 
contract 
Original's time t 
New time O 
Both mature on 
the same day 
Original's time T 
New time t 

# Forward Premium

* It is when the Forward Price is **GREATER** than the Spot Price
* This means that the **expectation** is that the **asset will depreciate in value**, which is why the forward that locks in a price is more expensive









**Futures**

# Forward VS Futures Contracts

|  |  |
| --- | --- |
| **Forwards** | **Futures** |
| Traded **Over the Counter**  **Customizable** Contracts | Traded on an **Exchange**  **Standardized** Contracts |
| Settled at **Expiration** | Settled **Daily (Marked to Market)** |
| Gains/Losses accumulate to large values increasing the chance of default; **high credit risk** | Gains/Losses are settled in small amounts daily, decreasing the chance of default; **low credit risk** |
| Harder to find counterparty due to customized contracts; **Low Liquidity** | Easy to find counterparty due to the standardized contracts; **High Liquidity** |
| No Price limits  Trades will continue **regardless** of price movements | Price Limits present  Trades will **stop** if price moves too much |

## Other Characteristics

* Futures are essentially Forwards that are traded on an exchange; **Exchange Traded Forwards**
* Futures are essentially Forwards with adjustments to **minimize credit risk**
* In practice, the frequent mark to marketing will **lead to price differences** between both
  + For calculation questions, we assume that these price differences are neglible
  + 

# Marked to Market

* Exchanges requires parties in a futures contract to start a **Margin account**:
  + **Initial Margin** → Initial amount to be deposited
  + **Maintenance Margin** → Minimum amount at any time
* Unlike a forward, the **value of a future contract is automatically calculated**
  + This calculation is **done once per day** at the end of the trading day using the **Settlement Price**
  + Any **change in the value** is **added/subtracted** from the margin account
  + The margin provides a **rough gauge** of the profit/loss of the contract
* If the value falls **below the Maintenance Margin**, then the investor needs to **top-up funds back till the initial margin** – known as a **Margin Call**
  + The rationale is to provide **additional buffer** should the price change again
  + If topped up to *just* the maintenance amount, any small change would trigger a margin call which is **inconvenient**
  + On the flip side, the **excess above the initial margin can be withdrawn** as profits
* At the end of each day, after any gains/losses/Margin calls, the **account earns interest**

# Futures Contracts

* Every Futures Contract has a **Notional Value/Contract Size**
* It is the amount of units of the underlying that will be traded under the contract
  + **Commodity Futures** → Contract Size is fixed
  + 
  + 
* All of the above values are calculated relative to the **Contract Size**

**Per Contract:**









* Note that the change in price is relative to the **LAST MARKED PRICE**
* The change in price is computed differently based on the perspective of the investor:
  + 
  + 





Note that the **multiplier** for an Index Futures contract is always 250.